Pre-Calculus

Team Members names:

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HOT AIR BALLOONS:

SURFACES OF REVOLUTION

Hot Air Surfaces of Revolution

Adapted from Robin Washam, Lynn Adsit, and Kim Schjelderup

A surface of revolution is a shape created by rotating a plane curve around its coordinate axis. For example, a standard cone is formed by rotating the line y = x around the x axis (remember the conics?). In our case, a balloon can be built by rotating the graph of a function about the x axis.

It is possible to approximate the shape of a balloon by using functions we studied this year. For example, using the function y = sin x over the domain of x = 0 to x = π creates a region that is a rough estimate for the shape of a balloon. The shape can then be modified by adding terms, multiplying x by a constant (a horizontal scale), multiplying sin x by a constant (a vertical scale), or multiplying sin x by x (compound functions). Two equations whose graphs look sort of like the outline of a balloon are (but are too small):

Trigonometric Function: $y = x \cdot \sin\left(\frac{\pi}{11}x\right)$ where x is in radians

Polynomial Function: $y = 3\left(\frac{x}{11}\right)^5 - 2\left(\frac{x}{11}\right)^4 + \left(\frac{x}{11}\right)^3 - \left(\frac{x}{11}\right)^2 + 2\left(\frac{x}{11}\right) - 1$

Window
X: [0, 6] Y: [0, 3]

Adding other functions such as y = cos x, composites of functions, along with scaling and transformations, can help produce a shape that looks even more like a hot air balloon. Creating a piece-wise function may also produce the shape you are looking for.

In this project, you will choose a function whose graph resembles the cross-section of a balloon. In order for the bottom to be open and to fit around the tube of the hot air source, the value of your function at x = 0 must be at least 4 cm, so let's say that at x = 0, y = 5 cm. **That is, your cross-section function must have a y-intercept of approximately $(0, 5)$.** You'll make the graph of the balloon cross-section 30cm wide and closed at the top, so that around $x = 100$ (the top of the balloon), y = 0. **So, the function has a zero at or near $(100, 0)$.** Between those two points, y > 0. In fact, to make the construction of the balloon easier, it would probably be a good idea that for a domain: 0 < x <100, the range should be $0 < y \leq 30ish$.

When you rotate that graph around the x axis, you will get the complete balloon shape. If you slice the balloon shape perpendicular to the x axis, the cross section is a circle. The perimeter of the circle at any point is 2πy (Why?). To construct the balloon from tissue paper, you will divide the surface of the balloon up into eight panels. At any point, each panel is approximately 1/8th of the perimeter of the circle cross section at that point. To summarize, at each point x along the graph of the function:

The **height** of the ***cross section*** curve is $y = f(x)$

The **circumference of the circle cross section** at that point is $C(x) = 2\pi y$

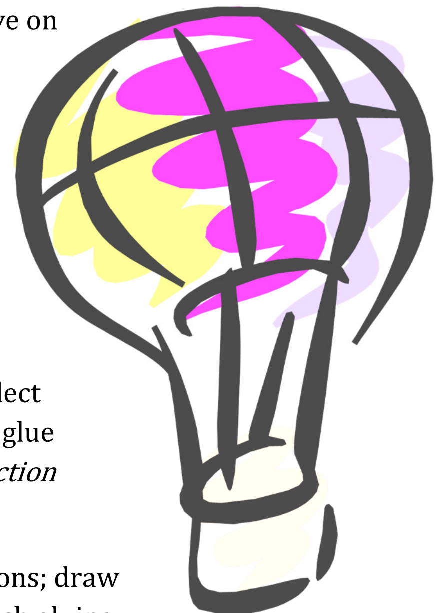
The **width of the panel** is $W(x) = \frac{C(y)}{8} = \frac{\pi y}{4}$

The length of each panel is a little more complicated than its width. Remember that you are covering the surface of the shape generated from a function that is not a straight line. If you go back to the original cross section, notice that the distance along the curve from any two points $P(x_1, y_1)$ to $Q(x_2, y_2)$ is not necessarily the same as the difference in the x values. If the two points are close enough together, a straight line is a reasonable approximation of the curve between the two points. You can calculate the length of the line segment from the two points $P(x_P, y_P)$ to $Q(x_Q, y_Q)$ by using the distance formula:

The distance from point P to point Q is $$PQ = \sqrt{\left(x_Q - x_P\right)^2 + \left(y_Q - y_P\right)^2}$$

Each panel of the balloon can be approximated by a stack of trapezoids for intervals of x from 0 to 100 in steps of 10 cm. The result will be much like making a connect-the-dot drawing of the balloon panel. Each trapezoid has a bottom base with width $W(x_P)$, the top base (parallel to the bottom base) width $W(x_Q)$, and the length is the distance PQ.

You want to make a tissue paper balloon large enough to fly with the hot air generated by a portable burner, so you will need to make a function that makes a balloon shaped curve on the domain $x = [0,100]$ and a range of $y = [0,30]$ on this domain.



Timeline for Project

School	**Balloon Project**
Day 1	Introduce the project; select groups of 2-3; begin to glue panels; create *cross section* equation
Day 2	Complete panel calculations; draw large panel template; finish gluing panels
Day 3	Trace and cut panels; glue panels and wire.
Day 4	Finish math work in packet; finalize balloons
Wed/Thurs (6/10-6/11)	Fly Balloons if weather permits (5th period)

Steps for designing a Hot Air Balloon

Step 0: Constructing the Panels

Some time while you are doing your calculations, you want to get your panels glued together so they are dry when you need to cut out your gores. You will need 8 panels to construct your balloon. Make these from colorful tissue paper. You will glue together tissue paper lengthwise (usually 3 pieces of tissue paper depending on the length of your paper) for one panel. Think about what kind of interesting design you can make!! They will need a total length of 58 " – 88" depending on the size of your cross section function. Begin by gluing 3 pieces of tissue paper together and you can add more later if you need it.

Step 1: Equation

In order for the bottom to be open and to fit around the tube of the hot air source, the value of your function at x = 0 must be at least 4 cm so let's say that at $x = 0cm$, $y = 5cm$ You'll make the graph of the balloon cross-section 30cm wide and is closed at the top, so that near $x = 100$ (the top of the balloon), $y = 0$. This gives us the following parameters for your function:

- $f(x)$ has a y-intercept at approximately (0,5)
- $f(x)$ has a zero at approximately (100,0)
- $0 < f(x) < 30ish$ for $0 < x < 100$

Explore **various types of equations** of the functions (piece-wise, polynomial, sinusoidal, etc.) that could represent the outline of the balloon.

- **Starting equation (that is close)**:____

- Record the **progressive alterations** you made to your starting equation and why.

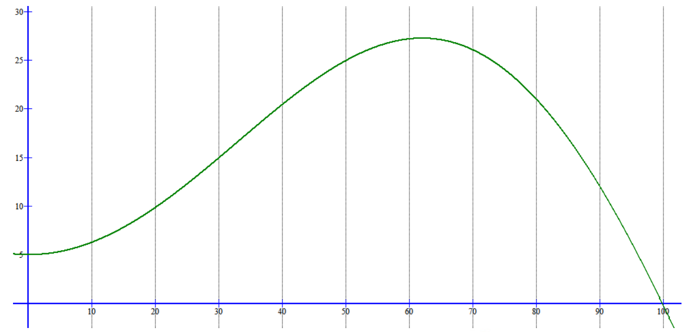
- **Describe your method** for choosing the "best" function. What were your criteria/thoughts?

Final Equation: ____ Teacher Okay: _______

Step 2: Drawing of Cross Section

You will find the values of your function at each 10cm interval; i.e. find the y-value associated with the corresponding x-inch value for your function. On your calculator, view the table for your cross-section function $f(x)$. **Fill in these values in 10 cm increments in the computation table on the next page.**

On the piece of graph paper at the end of your packet, draw your balloon outline using the cross section equation values from the computation table. Remember that the mouth of the full-sized balloon should have a diameter of approximately 5cm so that it will fit over the hot air source. It will look something like this:



Step 3: Panel Widths

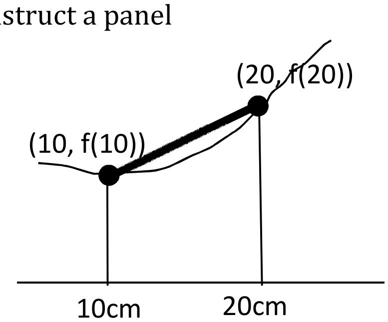
Your balloon is a three-dimensional object yet you will be making it using two-dimensional materials: tissue paper. In order to do this, you will craft 8 congruent panels which, when glued together, will form the three-dimensional balloon. A method of approximation that is useful is to use various circles centered one the x-axis and horizontally stacked along the axis to create the shape of the balloon; these are the *cross-sectional circles*. These *cross-sectional circles* show up nicely in the balloon to the right.

The **circumference** of each circle gives you the **girth** of the balloon at various points along the axis.

1. Calculate the circumference (to the nearest tenth) of the cross sectional circles at each 10cm mark along the $x-axis$ using the formula $C(x) = 2\pi y$ where y is the height of your cross-section function $f(x) = y$. ***Write these circumferences in your computation table.***
2. Now calculate 1/8 of the circumference and **fill in the values for $W(x) = \frac{2\pi y}{8}$ in your computation table.**

Step 4: Length of Balloon's Outer Edge = Panel Length

The panel must fit over the edge of the cross section piece, not along the main axis of the balloon; therefore, it is necessary to find an approximation of the arc length between each 10cm mark on the cross section. Using these arc lengths as tic marks on a new x-axis, you will construct a panel making sure that the 1/8 circumference is measured at each corresponding tic mark.



1. Begin by computing the distance between $(0, f(0))$ and $(10, f(10))$ using your cross-section function $f(x)$ and the distance formula $A(n) = \sqrt{(10-0)^2 + \left(f(10) - f(0)\right)^2}$.
2. Repeat this process to find the arc length between each 10cm point using the distance formula $A(n) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ and **record these in your computation table.**

Summary Formulas:

The **height** of the ***cross section*** curve is $y = f(x)$

The **circumference of the circle cross section** at that point is $C(x) = 2\pi y$

The **width of the panel** is $W(x) = \frac{C(y)}{8} = \frac{\pi y}{4}$

The approximate **arc length** between two points on $f(x)$ is $A(n) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

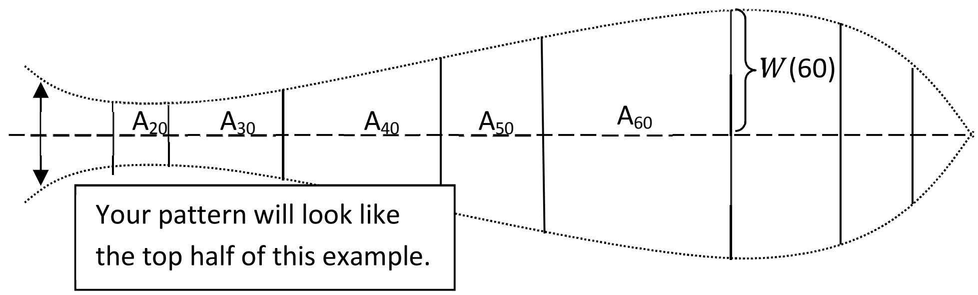
Computation Table

x *(Horizontal position in cross-section)*	$y = f(x)$ *(Vertical position in cross-section)*	$C(x) = 2\pi y$	$W(x) = \frac{2\pi y}{8}$	$A(n)$ Arc Length $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	½ $W(x)$
0				$A(0) = 0$	
10				$A(10) =$	
20				$A(20) =$	
30				$A(30) =$	
40				$A(40) =$	
50				$A(50) =$	
60				$A(60) =$	
70				$A(70) =$	
80				$A(80) =$	
90				$A(90) =$	
100				$A(100) =$	

Step 5: Creating the Panel Pattern:

We now need to make pattern for one gore. We will actually be making a pattern for a half of a gore because the gore is symmetrical.

1. Glue together several pieces of grid paper to make one long grid that is a little longer than the sum of all your $A(n)$ values. This will be used to make a pattern for half of a gore.
2. Mark off the arc lengths along the horizontal axis using the values of $A(n)$. Be careful!! *You will count off each new arc length from the previous tic mark.*
3. Next, at each tic mark, measure perpendicular to the folded edge the calculated scaled panel widths.
4. Connect the endpoints of these lengths with a smooth curve and cut out the pattern for *half* of the panel (called a gore).



Step 6: Constructing the Balloon

1. You need to make sure that your tissue paper panels are as long as your pattern and double the width of your pattern.
2. Cut out the eight panel pieces at the same time…these must be exactly the same size and shape. Use the paperclip method or the staple method shown in class. *The seam allowance should be about ½" beyond your pattern*.
3. You will then glue the panels together with a thin bead of glue along the edges. It is critical that you use a thin bead of glue smoothed out with a popsicle stick; too much glue will cause the panels to stick (and dry) together.
4. Also, use wire at the mouth of the balloon. Roll it up in the tissue paper and then glue. This will help keep the mouth open while getting the hot air.

Cross Section Graph

Hot Air Balloon

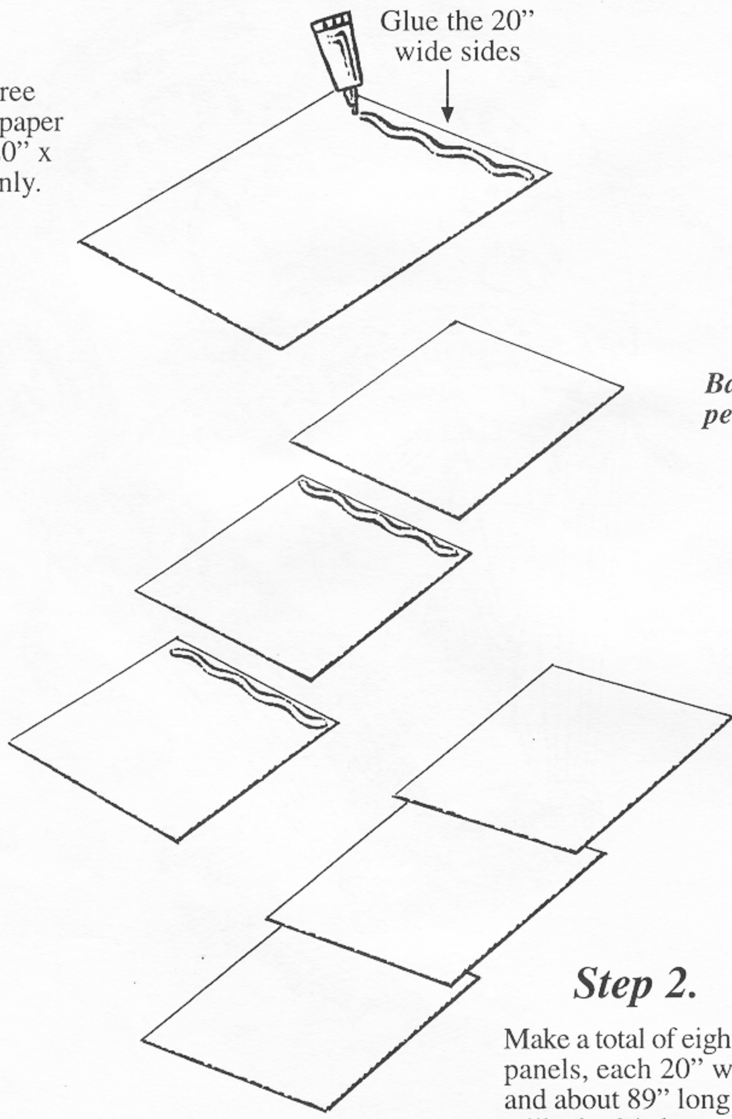
Making Your Balloon

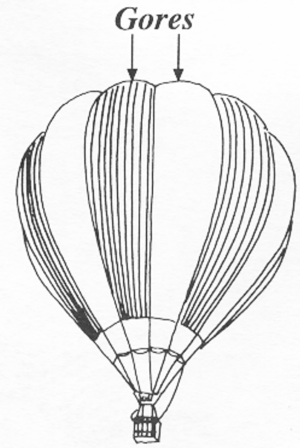
Your new Montgolfier is made of eight long panels called gores. Cutting these to the right shape and gluing them together will transform the paper into an amazing flying machine.

Making the gores

Step 1.

Make a gore by gluing three sheets of the blue balloon paper together. The sheets are 20" x 30." Glue the 20" sides only.





Balloons big enough to carry people are made of gores too, just larger.

Gluing Tips

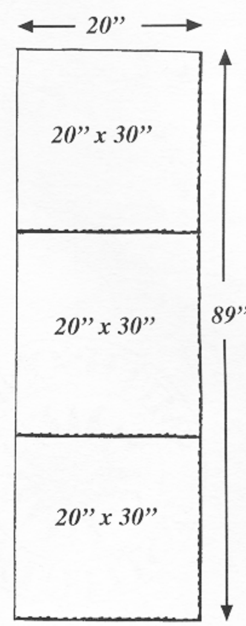
Use Elmer's Glue or another white craft glue.

We don't recommend glue sticks because the seams may come apart when the balloon is heated.

Use only a small bead of glue along one edge. It takes very little glue to make a strong seam. Overlap the seams about 3/8" to 1/2".

Let the glue dry completely before moving to the next step.

Make eight of these



Step 2.

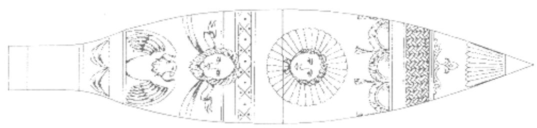
Make a total of eight gore panels, each 20" wide and about 89" long. This will take 24 sheets of the balloon paper.

Cutting the gores to the right shape

Step 3.

Tape or glue the balloon gore pattern together. Then lay one of the blue gores that you made on the floor. Trace the pattern outline with a pencil or pen.

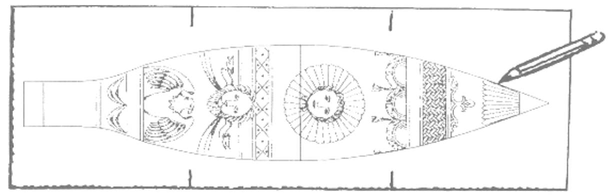
Three Piece Gore Pattern - 82.5" long



Tape or glue the pattern together

For Experienced Cutters Only

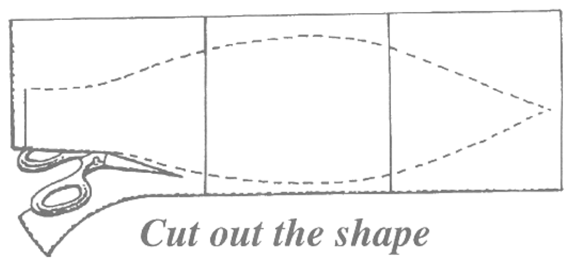
You can save time by stacking all eight gores, tracing the pattern outline on the top gore, and then cutting through all eight gores at one time. You will need to clip the edges together to keep the paper from shifting if you cut it this way.



Trace the pattern outline

Step 4.

With scissors cut the blue gore to the same shape as the outline.

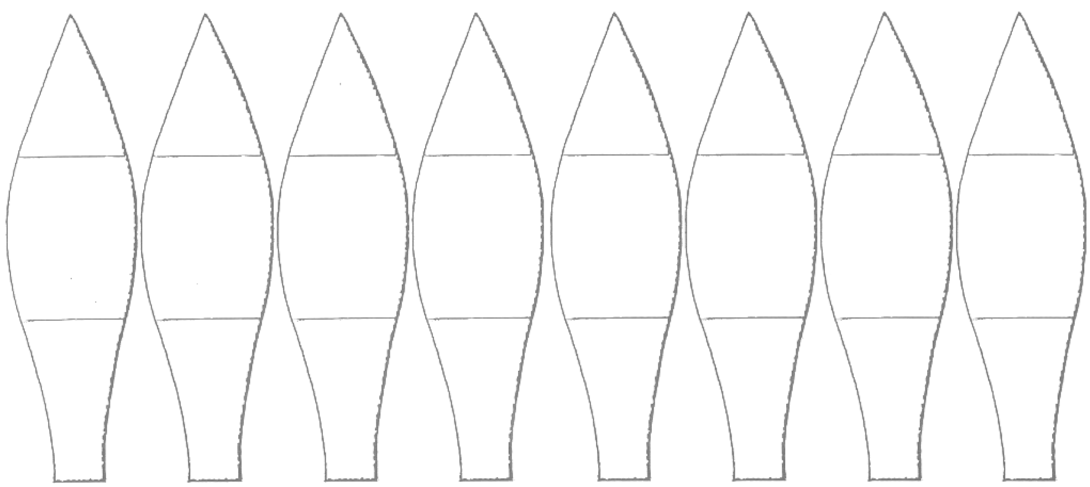


Cut out the shape

Step 5.

Repeat these steps for the other gores. When you're finished, you will have eight gores with the correct shapes to form a balloon.

Make 8 gores like this



Decorating decisions

It's best to put your decorations on the individual gores before you glue them together to make the balloon.

Important!!
Light balloons fly best

If you want your balloon to fly well, limit your decorations to ink and paint. Hot air balloons are very weight sensitive and every gram counts when flight time arrives.

Gluing the gores together

Step 6.

Lay the gores on the floor or a long table. Fold each of them lengthwise down the middle.

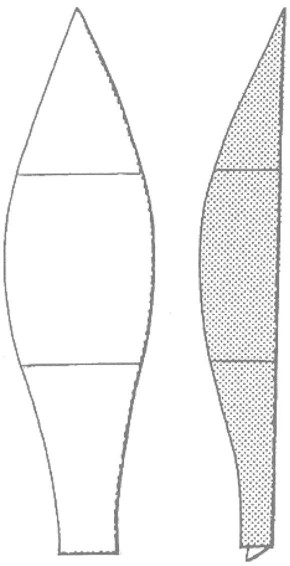
Step 7.

Run a thin bead of glue along one curved edge of a folded gore. Lay another gore over the first one, matching the edges. Press down to glue the gores together.

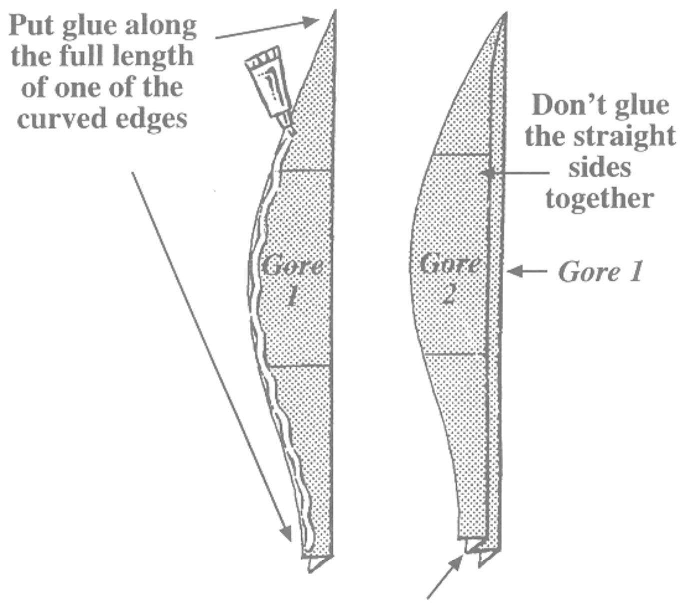
Step 8.

Continue adding one gore at a time, gluing all eight together like an accordion.

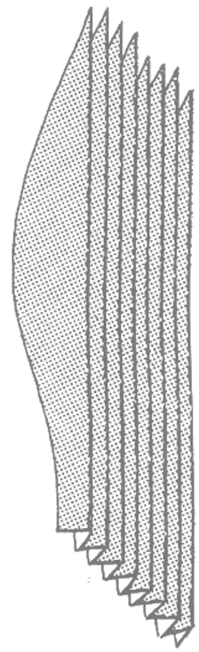
Fold the gores



Glue two gores together

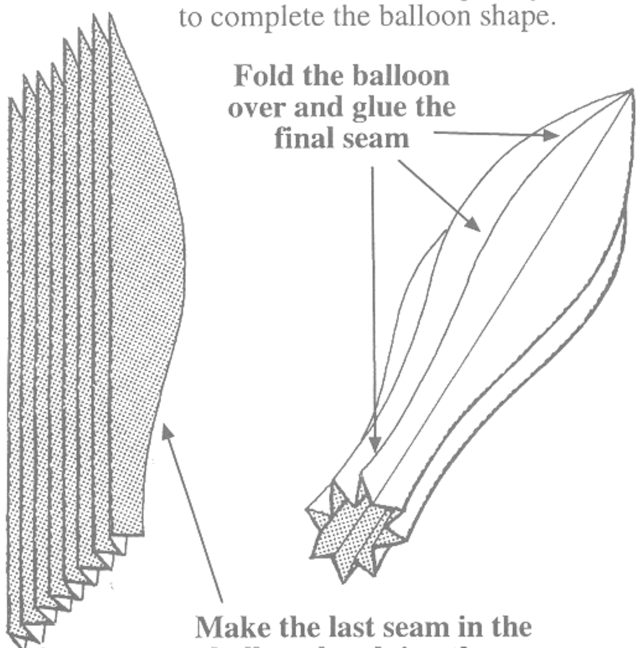


Glue all eight gores together



Step 9.

Glue the last two curved edges together to complete the balloon shape.



Gluing Tips

Use newspaper or other paper to protect floors and tables. Thin beads of glue will dry quicker and won't bleed through as much. Let each seam dry before you glue the next one. Don't let a seam that's drying stick to the gore under it.

Attaching the Wire Ring

With the wire in the kit make a loop about 20" wide. Hold the ends together with a small strip of tape.

Put the wire about 1/2" inside the bottom of the balloon. Fold the edge of the paper over the wire to keep it in place. Secure it with glue or small strips of tape about 4" apart.

Glue or tape wire ring inside the opening